Environmental Hydrology Chapter 8 Equations:

Specific Energy and Critical Discharge, Weirs, Flumes and Culverts, Routing Flows Through Rivers or Channels, Routing Flows Through Reservoirs, Stage-Storage Relationships, Detention or Residence Time, Sediment Storage in Management Ponds, Spillways

Specific Energy

If we take the bed of a channel as a datum the energy of the flow is:

$$E = \frac{v^2}{2g} + d \tag{8.1}$$

where E is called the *specific energy* (ft or m), d is the depth of flow (ft or m), v is the mean flow velocity (ft/s or m/s) and g is the gravitational constant. The term $v^2/2g$ is the velocity head and d is the head due to the depth of flow. Equation 8.1 can be related to the discharge by using the continuity equation (Equation 7.1, Chapter 7). Therefore:

$$E = \frac{\left(q/w\right)^2}{2gd^2} + d \tag{8.2}$$

Critical Discharge

When the specific energy is a minimum the flow is called critical flow and:

$$1 = \frac{v_c}{\sqrt{gd_c}} \tag{8.3}$$

The term $v_c/\sqrt{gd_c}$ is called the *Froude* number, F, and is equal to 1 when critical flow occurs. For all specific energy values, except when critical flow occurs, there are two possible depths of flow. For the deeper depth the flow will be stable and is called subcritical flow. For the shallower depth the flow will be unstable and is called supercritical flow. Flow prefers to be stable so at the first opportunity supercritical flow will try to become subcritical.

For a non-rectangular channel, the Froude number, F, can be related to the hydraulic depth, d_h, as follows:

$$F = \frac{v}{\sqrt{g d_h}} \tag{8.4}$$

where the hydraulic depth is the cross-sectional area divided by the top width.

Weirs, Flumes and Culverts

Application of specific energy concepts is useful when considering flow over weirs, spillways and riffles. It also has application where there is a change in the width of a channel. As flow passes over a hydraulic control structure it will often change from subcritical flow upstream of the structure to critical flow at the nappe of the flow across the structure. Downstream of the nappe the flow might initially be supercritical or might revert directly back to subcritical flow.

The discharge over a weir, q in cfs, can be estimated by using the equation:

$$q = C\, L\, H^{3/2} \tag{8.5}$$

where C is a weir coefficient, L is the weir length in feet, and H is the height of the flow above the riser crest in feet (Figure 8.4). For a broad-crested weir or a pipe a value of 3.1 may be used for C. For a pipe the weir length, L, will be the circumference of the pipe riser. For sharp-crested weirs, C is approximately 3.3 plus 0.4 times the ratio of the flow depth over the weir to the height of the weir.

A sharp-crested weir with a 90° "*V*" notch outlet, as shown in Figure 8.5, has the following discharge and head relationship:

$$q = 2.5\, H^{5/2} \tag{8.6}$$

where q is the discharge rate, ft^3/sec or cfs, and H is the head, or depth of water above the point of zero flow, ft. Calibration formulas for weirs with broad crests and different shapes are given by Brater and King (1976) or Bos et al. (1991).

Routing Flows Through Rivers or Channels

In the Muskingum method channel storage is the following linear function of inflow and outflow rates:

$$S = KO + KX(I - O) \tag{8.7}$$

where S is the storage in the reach, K and X are constants, and I and O are the simultaneous inflow and outflow, respectively.

For flood routing applications the Muskingum method is combined with the continuity equation, (equation 7.1) to give:

$$O_2 = O_1 + C_1\left(I_1 - O_1\right) + C_2\left(I_2 - I_1\right) \tag{8.8}$$

where the subscripts indicate the beginning and end of the time period t. The coefficients C_1 and C_2 are defined as follows:

$$C_1 = \frac{2\partial t}{2K(1-X) + \partial t} \tag{8.9}$$

and,

$$C_2 = \frac{\partial t - 2KX}{2K(1-X) + \partial t} \tag{8.10}$$

The coefficient K is called the storage constant and is approximated as the travel time of flow in the reach if streamflow records are not available for I and O. If X is zero then the procedure describes flow routing in an impoundment. If X is 0.5 the storage is a function of the average flow rate in the reach. A good account of the method is presented by Cudworth (1979).

Routing Flows Through Reservoirs

The change in reservoir water storage can be determined from the continuity equation as follows:

$$\delta S = \frac{(I_2 + I_1)}{2}\delta t - \frac{(O_2 + O_1)}{2}\delta t \tag{8.11}$$

where δS is the change in water storage, δt is the change in time between time t_1 and t_2, I_1 and I_2 are the inflow rates at times t_1 and t_2, and O_1 and O_2 are the outflow rates at times t_1 and t_2. Equation 8.11 is best solved by using a computer program or a spreadsheet.

The NRCS has related inflow and outflow rates, runoff volume, and storage volume as follows:

$$\frac{S}{V} = 1 - 2\left(\frac{q_{po}}{q_{pi}}\right) + 1.8\left(\frac{q_{po}}{q_{pi}}\right)^2 - 0.8\left(\frac{q_{po}}{q_{pi}}\right)^3 \tag{8.12}$$

where V is the runoff volume (area under the inflow hydrograph), S is the flood storage volume in the impoundment, q_{pi} is the peak inflow, and q_{po} is the peak outflow. This relationship should only be used for watershed areas of less than 250 acres. The relationship between the storage volume ratio and the flow rate ratio has been plotted in Figure 8.16.

Ward et al. (1979) developed a simple procedure which gives a conservative estimate of the required reduction in peak runoff rate and the storage volume:

$$\frac{S}{V} = 1 - \frac{q_{po}}{q_{pi}} \tag{8.13}$$

For a given ratio between the peak inflow and outflow rates, the required temporary storage will normally be between the values determined by the two methods.

Stage-Storage Relationships

Once a potential impoundment site has been located, a survey should be conducted and a topographic map prepared. For small structures with a capacity of less than 20 acre-ft storage, the contour interval should be 2–5 feet and the scale between 1:600 to 1:2400. The incremental volume between each stage elevation is determined from the equation:

$$V = \frac{(A_1 + A_2)(Z_2 - Z_1)}{2} \tag{8.14}$$

where A_1 and A_2 are the areas at elevations Z_1 and Z_2, and V is the volume of storage between elevations Z_1 and Z_2.

Detention or Residence Time

If the inflow and outflow hydrographs are approximated by triangular hydrographs, relationships can be established between the peak inflow and outflow rates, the base time of the inflow hydrograph, the volume of inflow, and the volume of flood storage in the impoundment.

From the geometry of a triangle, the base time of an inflow hydrograph, t_{bi}, is related to the inflow volume, V, and the peak inflow, q_{pi}, as follows:

$$V = 0.5 q_{pi}\, t_{bi} \tag{8.15}$$

If V is in acre-ft, q_{pi} is in cfs, and t_{bi} is in hrs, Equation 8.15 becomes:

$$V = 0.0413\, q_{pi}\, t_{bi} \tag{8.16}$$

A similar relationship can be established for the temporary storage:

$$S = 0.5\, t_{bi}(q_{pi} - q_{po}) \tag{8.17}$$

where S is the temporary storage volume, t_{bi} is the base time of the inflow hydrograph, q_{pi} is the peak inflow rate and q_{po} is the peak outflow rate. Equation 8.13 that was presented earlier is obtained by dividing Equation 8.17 by Equation 8.15.

If the storage volume is expressed in acre-feet, time in hours, and flow rates in cfs, then:

$$S = 0.0413\ t_{bi}(q_{pi} - q_{po}) \tag{8.18}$$

S may also be approximated by the equation:

$$S = 0.124\ t_d\ q_{po} \tag{8.19}$$

where t_d is the detention time in hours between the centroids of the hydrographs and q_{po} is the peak discharge rate. When Equations 8.18 and 8.19 are combined, q_{po} can be estimated from the relationship:

$$q_{p_o} = \frac{t_{bi}\ q_{pi}}{(3t_d + t_{bi})} \tag{8.20}$$

Sediment Storage in Management Ponds

Calculation of the sediment storage will depend on sediment discharges to a reservoir or sediment pond during the life of the pond and the trap efficiency of the structure. Methods presented in Chapter 9 can be used to calculate the amount of sediment that will reach the structure. The volume that the trapped sediment will occupy in the reservoir or pond can be calculated by using the following equation (Lara and Pemberton, 1963):

$$W = W_c P_c + W_m P_m + W_s P_s \tag{8.21}$$

where W_c, W_m, W_s, are the unit weights and P_c, P_m, P_s, are the fractions of clay, silt, and sand, respectively.

Spillways

Orifice flow occurs when flow is restricted by the size of the opening and is determined as:

$$Q = Ca\ (2gH)^{1/2} \tag{8.22}$$

where C is a coefficient depending on the orifice geometry, a is the cross-sectional area of the pipe, g is the gravitational constant, and H is the head in feet. For a sharp-edged orifice, a value of 0.6 may be used for C.

As the head continues to increase, the outlet begins to flow full and the flow is controlled by the outlet pipe. Pipe flow is determined by using the equation:

$$Q = \frac{a\ (2gH)^{1/2}}{(1 + K_e + K_b + K_c L)^{1/2}} \tag{8.23}$$

where K_e is an entrance loss coefficient, K_b is a correction factor for energy losses in bends and K_c is a friction factor. H is the head or difference in water elevations between the flow in the pond and at the outlet.