Environmental Hydrology Chapter 7 Equations:

Continuity and Manning's Equation, the Darcy-Weisbach Equation, Stream Power, the Value of Manning's n

Equation of Continuity

The discharge of a channel or river is given by the equation of continuity:

$$q = va \tag{7.1}$$

where; q is the discharge (ft^3/sec or m^3/sec), a is the cross-sectional area of the stream (ft^2 or m^2) obtained by simplifying the geometry and calculating the area or by drawing cross-sections to scale and measuring the area (see Chapter 14) and v is the average velocity of flowing water (ft/sec or m/sec).

Manning's Equation

For uniform flow in a channel the average velocity, v, can be estimated by Manning's equation:

$$v = \frac{1.5}{n} R^{2/3} S^{1/2} \tag{7.2}$$

where v is the velocity of flow (ft/s or m/s); n is Manning's roughness coefficient of the channel; S is the channel bed slope (ft/ft or m/m); and R is the hydraulic radius of the channel (ft or m) that is calculated as:

$$R = \frac{a}{P} \tag{7.3}$$

where P (ft or m), is the wetted perimeter of the channel cross-section (see Figure 7.1a)

If the flow is deep relative to the bed material size, Manning's n can be estimated by (Newbury and Gaboury, 1993):

$$n = 0.04 d_{50}^{1/6} \tag{7.4}$$

where d_{50} is the bed material size in meters. Care must be taken in using Equation 7.4 to estimate the bed roughness. If the depth of flow is very large compared to the d_{50}, the roughness might be less than estimated. However, if the depth of flow is similar or less than the d_{50}, the roughness will probably be greater than estimated.

A more difficult but common scenario is to have a known estimate of the discharge (perhaps by using a predictive methods described in Chapter 5) and be interested in evaluating the velocity and flow depth in an existing channel or river. An example of this scenario would be when an evaluation is being made of an upstream land use modification. Typically, an estimate would be made of the peak discharge associated with the land use modification, and then an analysis would be made to see if existing channels are able to convey the estimated peak discharge.

There are two approaches that could be used to evaluate this scenario. Equations 7.1 and 7.2 could be combined as follows in order to eliminate the flow velocity:

$$q = \frac{1.5}{n} aR^{2/3} S^{1/2} \tag{7.5}$$

The hydraulic radius and the cross-sectional area can then be written in terms of the depth of flow, d, that is now the only unknown. Unfortunately, Equation 7.5 is not easily solved as it contains complicated functions of depth.

The Darcy-Weisbach Equation

A more theoretically based equation than Manning's equation was developed by Darcy and Weisbach for determining head losses in pipes. When adapted for open channel flow it can be written as:

$$v^2 = \frac{8gRS}{f} \tag{7.6}$$

where f is the friction factor.

For straight uniform gravel-bed channels the friction factor can be estimated from the following form of the Colebrook-White equation:

$$f^{-1/2} = 2.03 \log\left(\frac{xR}{3.5\, d_{84}}\right) \tag{7.7}$$

where x can be approximated by:

$$x = 11.1 \left[\frac{R}{y}\right]^{-0.314} \tag{7.8}$$

where y is the perpendicular distance from the bed surface to the point of maximum velocity. This will be equivalent to the maximum flow depth if the width to depth ratio exceeds 2. All dimensions in these equations are in meters (m) , m^2, or m/s.

Stream Power

Channel modification might be accomplished by removing obstructions and decreasing the roughness coefficient (n) or resistance to water flow velocity or increasing the hydraulic gradient (s) by reducing meanders through channel straightening. If stream channel meanders are cut off and the channel straightened and cleaned, the roughness would be decreased and the slope increased. If prior to channel straightening the fall in 1,000 ft of a stream reach was 25.0 ft and this same fall occurred in 500 ft of a straightened channel, the gradient would increase from 0.025 to 0.05. Based on Manning's equation this would have the effect of increasing the velocity by a factor of 1.44 (the ratio of the square roots of 0.025 and 0.05). There is some danger in channel straightening in areas of highly erodible soil. When velocity and/or slope increase, there is more *stream power* (Bagnold, 1966) For any given portion of a stream cross-section, this may be written as:

$$S_p = \gamma v d S \tag{7.9}$$

where: S_p = stream power per unit measure of bed, here Kg per meter of width per second, γ = unit weight of water (includes temperature and sediment) kg/m^3, v = velocity of water, m/sec, d = depth in meter, S= slope, dimensionless fraction. See Chapter 12 for further discussion on this topic.

The Value of Manning's n

It is often difficult to quantify the influence of making channel modifications if Manning's n is obtained from Table 7.1. However, Chow (1959) presents information based on the work of Cowan (1956) that can be used for this purpose. The description of the procedure that follows is obtained from Chow (1959). The value of Manning's n is:

$$n = (n_0 + n_1 + n_2 + n_3 + n_4)m_5 \tag{7.10}$$

where n_0 is the value of n for a straight, uniform, smooth channel in natural materials, n_1 is a value added to n_0 to correct for the effect of surface irregularities, n_2 is a value for variations in shape and size of the channel cross-section, n_3 is a value for obstructions, n_4 is a value for vegetation and flow conditions, and m_5 is a correction factor for meandering of the channel.