Environmental Hydrology Chapter 6 Equations:

Lane's Channel Stability Model, Stream Orders, Law of Drainage Areas, Channel Patterns (Slope, Width, Depth, Velocity), Stream Stability and Sediment Transport, Meander Migration, Floodplains, and Streamways, Rosgen Stream Classification Method

Lane's Channel Stability Model

Lane's classic description of channel stability states that dynamic equilibrium exists between stream power and the discharge of bed-material sediment (Lane, 1955 as cited in Chang, 1998):

$$Q_s d \propto QS \tag{6.1}$$

where Q_s is the sediment discharge, d is the median sediment size, Q is the discharge and S is the bed slope.

Stream Orders

The ratio of the number of stream segments of a given order, N_n, to the number of segments of the next highest order, N_{n+1}, is called the bifurcation ratio, R_B:

$$R_B = \frac{N_n}{N_{n+1}} \tag{6.2}$$

Within a watershed the bifurcation ratio will change from one order to the next but will tend to be constant throughout the series. This observation forms the basis of the law of stream numbers (Horton, 1945) that states that the number of stream segments of each order form an inverse geometric sequence with order number, that is described mathematically as:

$$N_n = a_1 \exp^{-b_1 n} \tag{6.3}$$

where a_1 is a constant, and b_1 is ln R_B.

Two other drainage network laws have been developed. The law of stream lengths was also developed by Horton (1945) and is defined as follows:

$$L_n = a_2 \exp b_2 n \tag{6.4}$$

where a_2 is a constant, b_2 is ln R_L, and R_L is the stream length ratio. The stream length ratio is defined as:

$$R_L = \frac{L_{n+1}}{L_n} \tag{6.5}$$

where L_n and L_{n+1} are the average stream lengths of streams of order n and n+1, respectively.

Law of Drainage Areas

The law of drainage areas was developed by Schumm (1956) and is defined as follows:

$$A_n = a_3 \exp^{b_3 n} \tag{6.6}$$

where a_3 is a constant, b_3 is ln R_A, and R_A is the drainage area ratio:

$$R_A = \frac{A_{n+1}}{A_n} \tag{6.7}$$

where A_n and A_{n+1} are the average drainage areas of streams of order n and n+1, respectively.

The bifurcation ratio, stream length ratio, and drainage area ratio will in most cases fall within the ranges 3 to 5, 1.5 to 3.5, and 3 to 6, respectively. Also, Hack (1957) found that for many locations in the eastern U.S. the stream length was related to the drainage area as follows:

$$L = 1.4\, A^{0.6} \tag{6.8}$$

where L is the stream length in miles from the watershed boundary, and A is the watershed area in square miles.

Channel Patterns (Slope, Width, Depth, Velocity)

A relationship between the channel slope, bankfull discharge and channel pattern was reported by Leopold and Wolman (1957):

$$s = 0.0576\, Q^{-0.44} \tag{6.9}$$

where s is the streambed slope as a fraction, and Q is the bankfull (effective) stream discharge in ft^3/s (cfs).

Therefore, the channel width, depth, and mean velocity can be related to discharge as power functions:

$$W = a\,Q^b \tag{6.10}$$

$$d = c\,Q^f \tag{6.11}$$

$$v = k\,Q^m \tag{6.12}$$

Discharge is the product of the mean velocity times the cross-sectional area (w x d). Therefore, a x c x k equals 1 and b + f + m equals 1.

Stream Stability and Sediment Transport

Shear Stresses and Tractive Force

The mean boundary shear stress, τ_0 exerted by the flow on the bed can be estimated from:

$$\tau_o = \rho g R s = \gamma R s \tag{6.13}$$

where ρ is the density of water, γ is the specific weight of water, R is the hydraulic radius and s is the bed slope. A more general term called the tractive force, or average shear stress in a reach, can be related to the depth of flow and the average slope of the water surface as follows:

$$T = 62.4ds \tag{6.14}$$

Where T is the mean shear stress or tractive force (lb/ft^2), d is the flow depth in ft, s is the slope of the water surface as a fraction, and 62.4 lb/ft^3 is the specific weight of water. The mean particle size that can be moved at incipient motion by a certain tractive force can then be determined by the relationship expressed as:

$$d_{50} = cT \tag{6.15}$$

where T is the tractive force (kg force/m^2), the mean diameter d_{50} is in centimeters (cm), and c is a unit conversion factor that is also a function of the magnitude of the tractive force. If the tractive force is larger than 1 kg force/m^2 then c is equal to one. The tractive force in kg force/m^2 can be obtained by expressing the depth in meters and the specific weight as 1000 kg/m^2 in Equation 6.14.

Sediment Transport Rate

The daily sediment transport rate, S (metric tons/day), can be estimated by a power function of discharge, Q (m^3/s) (Nash, 1994):

$$S = aQ^b \tag{6.16}$$

where a and b are empirical coefficients. Nash (1994) reported values of b between 1.23 and 3.02 with 38 out 55 watersheds he evaluated exhibiting values between 1.4 and 1.9.

Meyer-Peter-Muller Equation

Bedload can be determined by the Meyer-Peter-Muller equation (Chanson, 1999):

$$\left[\frac{q_s}{\left(1.65 g d_{50}{}^3\right)^{1/2}}\right] = \left[\frac{4\tau_o}{1.65\rho g d_{50}} - 0.188\right]^{3/2} \tag{6.17}$$

where q_s is the volumetric bedload discharge (m^3/s per unit width), d_{50} is the mean particle size of the bed material (m), τ_0 is the shear stress ($kg/m\text{-}s^2$), ρ is the density of water (kg^3/m^3), the specific gravity of the sediment is 2.65, and g is the gravimetric constant (m/s^2).

Meander Migration, Floodplains, and Streamways

Many empirical equations have been developed to describe bankfull (effective discharge) channel geometry. One such equation by Williams (1986) relates meander beltwidth (B, m) and the bankfull width (W, m) as follows:

$$B = 4.3\ W^{1.12} \tag{6.18}$$

Equation 6.18 was based on 153 data points from rivers around the world and the correlation coefficient (r) for the equation is 0.96. Belt width and bankfull width data for 47 of the locations are presented in the paper by Williams (1986). Based on an analysis of this data they developed the following equation to obtain the streamway width, S_w equation:

$$S_w = 6.0\ W^{1.12} \tag{6.19}$$

Where, S_w and W are now expressed in feet. Then based on regional curves for the Eastern United States that relate channel width to drainage area, DA (mi^2) they obtained the relationship:

$$S_w = 120\ DA^{0.43} \tag{6.20}$$

Rosgen Stream Classification Method

The channel sinuosity is an index of channel pattern, determined from a ratio of stream length divided by valley length; or estimated from a ratio of valley slope divided by channel slope. The entrenchment ratio (ER) is the ratio of the flood-prone area width (W_{fpa}) divided by bankfull channel width (W_{bkf}):

$$ER = \frac{W_{fpa}}{W_{bkf}} \qquad (6.21)$$

The bankfull width (W_{bkf}) is the width of the stream channel at the bankfull stage elevation in a riffle section. The mean depth (d_{bkf}) is the depth of the stream channel at the bankfull stage elevation in a riffle section. The width to depth ratio (W-D) is:

$$W - D \text{ Ratio} = \frac{W_{bkf}}{d_{bkf}} \qquad (6.22)$$

The maximum depth (d_{mbkf}) is the depth of the bankfull channel cross-section, or vertical distance between the bankfull stage and thalweg elevations, in a riffle section. The flood-prone area width (W_{fpa}) is measured at an elevation that is twice the maximum depth at the location that the maximum depth was determined.