Environmental Hydrology Chapter 5 Equations:

100-Year Recurrence Interval Discharge, NRCS Curve Number Method, Graphical Peak Discharge Method, The Rational Equation, USGS Empirical Regression Models, Unit Hydrograph Methods, Ellipse Equation for Subsurface Drainage Flows

100-Year Recurrence Interval Discharge

The influence of watershed shape was found to be significant in several studies by the USGS where they statistically related climatic and watershed properties to stream discharge data. The 100 year recurrence interval discharge, Q_{100}, (m^3/s), is related to the elevation E (m), and the basin shape factor SH defined as the drainage area divided by the square of the main channel length, as follows:

$$Q_{100} = 0.471 A^{0.715} E^{0.827} SH^{0.472} \tag{5.1}$$

NRCS Curve Number Method

Rainfall excess (volume of runoff) could be determined by using one of the infiltration equations described in Chapter 3, but this approach has only been incorporated in a few hydrologic computer models. The most commonly used method, in the United States, is the NRCS curve number procedure (NRCS, 1972). In this approach infiltration losses are combined with surface storage by the relationship:

$$Q = \frac{(P - I_a)^2}{(P - I_a + S)} \tag{5.2}$$

where Q is the accumulated runoff or rainfall excess in inches, P is the rainfall depth in inches, and S is a parameter given by:

$$S = \frac{1000}{CN} - 10 \tag{5.3}$$

where CN is known as the curve number. The term I_a is the initial abstractions in inches and includes surface storage, interception, and infiltration prior to runoff. The initial abstractions term I_a is commonly approximated as $0.2S$ and Equation 5.2 becomes:

$$Q = \frac{(P - 0.2\,S)^2}{(P + 0.8\,S)} \tag{5.4}$$

Graphical Peak Discharge Method

The Graphical Peak Discharge method (NRCS, 1986) was developed for application in small rural and urban watersheds. It was developed from hydrograph analyses with TR-20 *Computer Program for Project Formulation - Hydrology* (NRCS, 1973b) and has seen widespread application. The peak discharge equation is:

$$q = q_u\, A\, Q\, F \tag{5.5}$$

where q is the peak discharge (runoff rate, cfs), q_u is the unit peak discharge (cfs per square mile per inch of runoff, csm/in), A is the drainage area (mi^2), Q is the runoff depth (in.) based on 24 hours, and F is an adjustment factor for ponds and swamps. The method depends on the NRCS curve number method (Equations 5.2, 5.3, 5.4) to obtain Q and the necessary information to determine q_u. The unit peak discharge, q_u, requires knowledge of the time of concentration and the initial abstraction, I_a, from Equations 5.2 and 5.4.

Time of Concentration and Lag Time

Numerous equations have been developed for determining the *time of concentration* and *lag time*. The time of concentration is the time it takes water to travel along the hydraulic length while the lag time is the average of the flow times from all locations in the watershed. One recommended approach is to use the NRCS lag equation:

$$t_L = \frac{L^{0.8}(S+1)^{0.7}}{1900Y^{0.5}} \quad (50 < CN < 95) \tag{5.6}$$

where t_L is the lag time in hours, L is the hydraulic length of the watershed in feet, S is a function of the NRCS curve number (see Equation 5.2), and Y is the average land slope in percent. The lag time is related to the time of concentration as follows

$$t_L = 0.6 t_c \tag{5.7}$$

Lag time is an estimate of the average flow time for all locations on a watershed. The coefficient of 0.6 in Equation 5.7 accounts for the fact that the average flow time will be 0.45 to 0.65 of the maximum flow time, depending on the watershed shape. Often the average land slope is approximated as the slope along the hydraulic length.

The TR-55 computer model (NRCS, 1986) calculates the time of concentration as the sum of the travel times due to sheet flow, shallow concentrated flow, and open channel flow along the hydraulic length. For sheet flow the travel time, t_t, is estimated as:

$$t_t = \frac{0.007(nL)^{0.8}}{(P_2)^{0.5}(S)^{0.4}} \qquad (5.8)$$

where n is Manning's roughness factor, L is the flow length in feet, P_2 is the 24-hour 2-year return period rainfall in inches, and S is the land slope or slope of the hydraulic grade line in ft/ft. For shallow concentrated flow:

$$t_t = \frac{L}{K_1(S)^{0.5}} \qquad (5.9)$$

where K_1 is 58084 for unpaved surfaces and 73182 for paved surfaces.

For open channel flow:

$$t_t = \frac{nL}{5364(a/p_w)^{0.67}(S)^{0.5}} \qquad (5.10)$$

where a is the cross-sectional area of flow in square feet, L is the channel length in feet, and p_w is the wetted perimeter in feet. Equations 5.8, 5.9, and 5.10 were developed based on Manning's equation (see Chapter 7). An approximate simplification of equations 5.9 and 5.10 is obtained by determining the flow velocity from the equation:

$$v = K_2(S)^{0.5} \qquad (5.11)$$

where v is the flow velocity (ft/s) and a is a coefficient that is based on Manning's roughness values and the depth of flow.

The Rational Equation

In the United States, the Rational Equation is the most widely used empirical method:

$$q = 1.008\,\mathrm{CiA} \qquad (5.12)$$

where *q* is the peak flow (cfs), *C* is an empirical coefficient, *i* is the average rainfall intensity (in./hr) during the time of concentration, *A* is the drainage area (acres), and 1.008 is a unit conversion factor (1 in./hr × 1 ft / l2 in × 1 hr/3600 sec × 1 acre × 43,560 ft2/1 acre). The unit conversion factor of 1.008 is usually approximated as 1.0 because of the uncertainties associated with determining each of the other equation parameters. The time of concentration is the time it takes flow to move from the most remote point on a watershed to the outlet of the watershed. This longest flow path is called the hydraulic length.

A commonly used time of concentration method that is used in conjunction with the Rational method is the following equation that was developed by Kirpich (1940):

$$t_c = 0.00778\, L^{0.77} S^{-0.385} \tag{5.13}$$

where *L* is the hydraulic length (maximum length) in feet, *S* is the mean slope along the hydraulic length expressed as a fraction, and t_c is the time of concentration in minutes.

USGS Empirical Regression Models

Most of the USGS regression equations have the following common form:

$$Q_{RI} = a(DA)^b (S)^c (P - X)^d (13 - BDF)^e (I)^f (U)^g (STR + 1)^h (E)^i (Q_{rt}.)^j \ldots\ldots \tag{5.14}$$

where Q_{RI} is the peak discharge for a particular recurrence interval (cfs), DA is the drainage area, S is the channel bedslope, P is an index of precipitation (often the 2-year 24-hour precipitation depth or the mean annual depth in inches), BDF is the Basin Development Factor for urban areas, I is the percent impervious area, U is an index (or indices) of land use (percent forest etc), STR is the percentage of the contributing drainage area occupied by lakes, ponds, and wetlands, E is a function the elevation (1000's of ft for Central Coastal Region of California), where Q_{rT} is the peak discharge for a return period T years and an equivalent rural watershed and X, a,b,c, d, etc are empirical coefficients that are a function of the recurrence interval and region. All equations contain the drainage area, DA, and then just a few (some times none) of the other variables.

Unit Hydrograph Methods

The basic factors which need to be determined in order to develop a unit hydrograph are the time to peak, the peak flow and the shape of the hydrograph. The time to peak is defined as:

$$t_p = t_L + D/2 \tag{5.15}$$

where t_L is the lag time and *D* is the time increment of the rainfall excess. A plot of the rainfall excess versus time for each time increment, *D*, is called a hyetograph.

The peak flow of the unit hydrograph can be estimated by the equation (NRCS,1972):

$$q_p = \frac{484\,A}{t_p} \tag{5.16}$$

where q_p is the peak flow in cfs (per inch of runoff), *A* is the watershed area in square miles and t_p is the time to peak in hours.

Ellipse Equation for Subsurface Drainage Flows

For steady state conditions where the water table and rainfall or irrigation rate does not change with time, the ellipse equation can he used to determine a drain spacing and flow rate:

$$q = \frac{4K(b^2 - d^2)}{S} \tag{5.17}$$

where *K* is the saturated hydraulic conductivity (ft/day), *q* is the flow rate (ft^2/day) into the drain per foot length of drain, *S* is the drain spacing in feet, *b* is the height (feet) at the midpoint between the drains of the water table above an impeding layer, and *d* is the height (feet) of the drains above the impeding layer.

If the flow rate, *q*, is set equal to *Si*, where *i* is the drainage rate in ft/day, then Equation 5.17 can be written as:

$$S = \left[\frac{4K(b^2 - d^2)}{i}\right]^{1/2} \tag{5.18}$$