Environmental Hydrology Chapter 4 Equations:

Evaporation Process, Measuring Evaporation or Evapotranspiration, Weather Data Sources and Preparation, Estimating Evaporation or Evapotranspiration, Crop Actual Evapotranspiration

Evaporation Process

Fick's First Law of Diffusion

Fick was one of the first people to quantify the movement of molecules from a region of higher concentration to a region of lower concentration (Nobel, 1983), such as water molecules moving from a water surface into air. He developed Fick's first law of diffusion:

$$J_j = -D_j \frac{\partial c_j}{\partial x} \tag{4.1}$$

where Jj (the flux density) is the amount of species *j* crossing a certain area per unit time and is typically expressed in units such as moles of particles per m^2 per second. D_j is the diffusion coefficient of species *j* (analogous to resistance in electrical circuits). The term $\partial c_j / \partial x$ represents the concentration gradient of species *j* and is the driving force that leads to molecular movement (Nobel, 1983). The negative sign indicates that the direction of flow is from high to low concentration.

Measuring Evaporation or Evapotranspiration

Pan Evaporation

Pan evaporation data can be used to estimate actual evapotranspiration of a reference crop using the following equation (Jensen et al., 1990):

$$E_{tr} = k_p E_{\mathrm{pan}} \tag{4.2}$$

where k_p is a crop or pan coefficient. Many k_p values have been determined in previous studies but it is important that the study have a similar climate (humid vs. arid) and use the same pan (i.e., class A pan) with similar nearby surfaces and placement in relation to wind barriers at our site of interest.

Soil Water Depletion

AET from a crop can be estimated by observing the change in soil water over a period of time. The average rate of *ET* in mm/d between sampling dates (denoted Δ*t*) can be calculated using the following equation (Jensen et al., 1990):

$$E_t = \frac{\Delta \mathrm{SM}}{\Delta t} = \frac{\sum_{i=1}^{n_{rz}} (\theta_1 - \theta_2)_i \Delta S_i + I - D}{\Delta t} \tag{4.3}$$

where
- E_t = actual evapotranspiration in mm/d
- ΔSM = change in soil water content
- Δt = time between sampling dates
- n_{rz} = number of soil layers in the effective root zone
- ΔS_i = the thickness of each soil layer in mm
- θ_1 = volumetric water content of soil layer *i* on the first sampling date (m^3/m^3)
- θ_2 = volumetric water content of soil layer *i* on the second sampling date (m^3/m^3)
- I = infiltration (rainfall - runoff) during Δt (mm)
- D = drainage below the root zone during Δt (mm)

Water Balance

The water balance approach is generally used on large areas such as watersheds. The inflows and outflows are determined from streamflow and precipitation measurements and the difference between inflow and outflow over a relatively long period of time, such as a season, is a measure of evapotranspiration. The equation is:

$$\mathrm{AET} = \mathrm{P} - \mathrm{Q} \pm \Delta\mathrm{G} \pm \Delta\theta \tag{4.4}$$

where P is the precipitation depth, AET is actual ET, Q is runoff depth, ΔG is ground-water inflow or outflow, and Δθ is soil water change. For the short term, ΔG and Δθ should be measured, but over a period of years they become insignificant and can be dropped, so the equation becomes:

$$\mathrm{AET} = \mathrm{P} - \mathrm{Q} \tag{4.5}$$

Alternatively, Equation 4.6 can be used to compute saturation vapor pressure, e_s in kPa if temperature, T, is in degrees Celsius:

$$e_s = \exp\left[\frac{16.78\,T - 116.9}{T + 237.3}\right] \tag{4.6}$$

This equation is useful if we would like to write a computer program to compute some of these values. The equation is valid for temperatures ranging from 0 to 50°C.

Weather Data Sources and Preparation

Actual Vapor Pressure

Actual vapor pressure, e_d, is the vapor pressure of the air. Unlike saturation vapor pressure, actual vapor pressure cannot be determined simply by knowing the temperature of the air. To determine e_d we need to know the air temperature and either the relative humidity or the dewpoint temperature of the air. The following equation can be used to find actual vapor pressure:

$$e_d = e_s \times \frac{\mathrm{RH}}{100} \tag{4.7}$$

where: e_d = actual vapor pressure
e_s = saturation vapor pressure
RH = relative humidity in percent

Vapor Pressure Deficit

Many methods exist for calculating the vapor pressure deficit ($e_s - e_d$) (see Jensen et al., 1990). Three methods for calculating vapor pressure deficit are shown here.

Method 1. Saturation vapor pressure at mean temperature minus saturation vapor pressure at dewpoint temperature, this can be written as:

$$(e_s - e_d) = e_{s_{(T_{\mathrm{avg}})}} - e_{s_{(T_d)}} \tag{4.8}$$

where e_d = actual vapor pressure
e_s = saturation vapor pressure
T_{avg} = mean temperature for time period of interest
T_d = mean dewpoint temperature for time period of interest

Method 2. The vapor pressure deficit can be estimated from the saturation vapor pressure at the mean temperature times the quantity one, minus the relative humidity expressed as a proportion or:

$$(e_s - e_d) = e_{s_{(T_{avg})}} \left(1 - \frac{\mathrm{RH}}{100}\right) \tag{4.9}$$

This equation is obtained by writing equation 4.7 as:

$$e_{s_{(T_s)}} = e_{s_{(T_{avg})}} \frac{\mathrm{RH}}{100} \tag{4.10}$$

and then substituting equation 4.10 into equation 4.8.

Method 3. The mean of saturation vapor pressure at the maximum and minimum temperatures minus the saturation vapor pressure at the dewpoint temperature determined early in the day, typically at 8 a.m.

$$(e_s - e_d) = \frac{e_{s(T_{\max})} + e_{s(T_{\min})}}{2} - e_{s(T_{d8\mathrm{am}})} \tag{4.11}$$

The most likely scenario is that R_s has been measured and we need R_n to use Penman's method or a similar method. It is possible to estimate R_n from R_s since R_n is the net short-wave minus the long-wave components of the radiation.

$$R_n = (1 - \alpha) R_s \downarrow - R_b \uparrow \tag{4.12}$$

where R_b is the net outgoing thermal radiation in MJ/m^2/d and α is the albedo or short-wave reflectance, which is dimensionless. The arrows in Equation 4.12 serve as reminders that *Rs* is incoming and *Rb* is outgoing. The short-wave reflectance or albedo, α is typically set equal to 0.23 for most green field crops with a full cover (Jensen et al., 1990). Since we know R_s and α, R_b is all that is needed. Equation 4.13 can be used to calculate this value, in units of MJ/m^2/d

$$R_b = \left[a \frac{R_s}{R_{so}} + b \right] R_{bo} \tag{4.13}$$

The coefficients *a* and *b* are determined for the climate of the area of interest. For humid areas, $a = 1.0$ and $b = 0$; for arid areas, $a = 1.2$ and $b = -0.2$; and for semi-humid areas $a = 1.1$ and $b = -0.1$. R_{so} is the solar radiation on a cloudless day (in units of MJ/m^2/d) based on the site's latitude. R_{bo} can be computed from Equation 4.14.

$$R_{bo} = \epsilon \, \sigma T^4 \tag{4.14}$$

where the Stefan-Boltzmann constant, σ, is 4.903×10^{-9} MJ/m^2/d/K^4, and T is the mean temperature for the period of interest in Kelvin (273 + T in °C). The term , is the net emissivity and is calculated using the Idso-Jackson equation (Equation 4.15) with T in Kelvin.

$$\epsilon = -0.02 + 0.261 \ \exp[-7.77 \times 10^{-4} (273 - T)^2] \tag{4.15}$$

Extrapolating Wind Speed

Wind is typically slower at the ground surface and the speed increases with height. Most methods for estimating *ET* that require wind speed specify at what height the wind speed should be recorded. However, in practice the data have sometimes been recorded at other heights. To estimate the wind speed, u_2, at height z_2, knowing the wind speed u_1 at height z_1, Equation 4.16 can be used (Allen et al., 1989).

$$\frac{u_1}{u_2} = \frac{\ln[z_1 - 0.67h_c] - \ln[0.123h_c]}{\ln[z_2 - 0.67h_c] - \ln[0.123h_c]} \tag{4.16}$$

where h_c is the height of the vegetation, $0.67h_c$ is the height where the wind velocity approaches zero (known as the roughness height), and $0.123h_c$ is the surface roughness. The variables h_c, z_I, and z_2 are expected to have the same units, then u_2 will have identical units to u_1.

Estimating Evaporation or Evapotranspiration

Evaporation from Open Water

Monthly evaporation from lakes or reservoirs can be computed using the empirical formula developed by Meyer (1915) but based on Dalton's Law (1802)(Harrold et al., 1986).

$$E = C(e_s - e_d)\left(1 + \frac{u_{25}}{10}\right) \tag{4.17}$$

where
- E = evaporation in inches/month
- e_s = saturation vapor pressure (inches of Hg) of air at the water temperature 1 foot deep
- e_d = actual vapor pressure (inches of Hg) of air = $e_{s\ (\mathrm{air\ T})} \times \mathrm{RH}$
- u_{25} = average wind velocity (mi/hr) at a height of 25 feet above the lake or surrounding land areas
- C = coefficient that equals 11 for small lakes and reservoirs and 15 for shallow ponds

SCS Blaney-Criddle Method

Blaney and Criddle assumed that mean monthly air temperature and monthly percentage of annual daytime hours could be used instead of solar radiation to provide an estimate of the energy received by the crop. They defined a monthly consumptive use factor, f, as:

$$f = \frac{tp}{100} \tag{4.18}$$

where t is the mean monthly air temperature in °F (avg. of daily maximum and minimum) and p is the mean monthly percentage of annual daytime hours. The 100 in the divisor converts p from a percentage to a fraction. Once f is computed for each month, then the actual ET for the season is computed by Equation 4.19:

$$U = K\sum_{i=1}^{n} f_i \tag{4.19}$$

where K is the seasonal consumptive use coefficient for a crop with a normal growing season, n is the number of months in the season, and U is the seasonal consumptive use in inches/season.

If we have monthly consumptive use coefficients available for the specific crop and location, then monthly consumptive use (u) can be computed as follows:

$$u = k\frac{tp}{100} \tag{4.20}$$

where k is an empirical coefficient and u is the monthly consumptive use in inches/month.

Jensen-Haise Alfalfa-Reference Radiation Method

The Jensen-Haise method is termed a radiation method because solar radiation is needed in the equation to incorporate the recognized link between a source of energy and evapotranspiration. Jensen and Haise used over 3000 observations of actual evapotranspiration determined by soil sampling and statistically related R_s to E_{tr} as shown in Equation 4.21 (Jensen and Haise, 1963).

$$E_{tr} = \frac{C_T(T - T_x)R_s}{\lambda} \tag{4.21}$$

where E_{tr} = reference evapotranspiration in mm/d
C_T = temperature coefficient (Equation 4.22)
λ = latent heat of vaporization in MJ/kg (Equation 4.26)
R_s = solar radiation received at the earth's surface on a horizontal surface, MJ/m^2/d
T = mean temperature for a 5-day period, °C
T_x = intercept of the temperature axis (Equation 4.25), °C

The temperature coefficient can be calculated as follows:

$$C_T = \frac{1}{C_1 + 7.3\,C_H} \tag{4.22}$$

and C_1, which is needed to calculate C_T, can be calculated from:

$$C_1 = 38 - \frac{2(H)}{305} \tag{4.23}$$

where H is the elevation above sea level in meters.

C_H, which is also needed for Equation 4.22, is calculated as follows:

$$C_H = \frac{5.0\text{ kPa}}{(e_2 - e_1)} \tag{4.24}$$

where e_2 and e_1 are the saturation vapor pressures in kPa at the mean maximum and mean minimum temperatures, respectively, for the warmest month of the year in an area.

$$T_x = -2.5 - 1.4\,(e_2 - e_1) - \frac{H}{550} \tag{4.25}$$

$$\lambda = 2.501 - 2.361 \times 10^{-3}\, T \tag{4.26}$$

where λ is the latent heat of vaporization (MJ/kg) and T is temperature in °C (Harrison, 1963).

Thornthwaite Method

Thornthwaite found that evapotranspiration could be predicted from an equation of the form:

$$E_{tp} = 16\left[\frac{10T}{I}\right]^{a} \tag{4.27}$$

where E_{tp} = monthly ET in mm
T = mean monthly temperature in °C
a = location dependent coefficient described by Equation 4.29
I = heat index described by Equation 4.28

In order to determine **a** and monthly ET, a heat index I must first be computed.

$$I = \sum_{j=1}^{12}\left[\frac{T_j}{5}\right]^{1.514} \tag{4.28}$$

where T_j is the mean monthly temperature during month j (°C) for the location of interest.
Then, the coefficient **a** can be computed as follows:

$$a = 6.75 \times 10^{-7} I^3 - 7.71 \times 10^{-5} I^2 + 1.792 \times 10^{-2} I + 0.49239 \tag{4.29}$$

Penman's Method

Penman (1948) first combined factors to account for a supply of energy and a mechanism to remove the water vapor from the immediate vicinity of the evaporating surface. We should recognize these two factors as the essential ingredients for evaporation. Penman derived an equation for a well watered grass reference crop:

$$E_{tp} = \frac{\dfrac{\Delta}{\Delta+\gamma}(R_n - G) + \dfrac{\gamma}{\Delta+\gamma} 6.43\,(1.0 + 0.53\,u_2)(e_s - e_d)}{\lambda} \tag{4.30}$$

where E_{tp} = potential evapotranspiration in mm/day
R_n = net radiation in MJ/m/d
G = heat flux density to the ground in MJ/m/d

λ = latent heat of vaporization computed by Equation 4.26 in MJ/kg
u_2 = wind speed measured 2 m above the ground in m/s
Δ = slope of the saturation vapor pressure-temperature curve, kPa /°C
γ = psychrometric constant, kPa/°C
$e_s - e_d$ = vapor pressure deficit determined by Method 3; kPa

The slope of the saturation vapor pressure-temperature curve, Δ, can be computed knowing the mean temperature as follows:

$$\Delta = 0.200\,[0.00738\,T + 0.8072]^7 - 0.000116 \tag{4.31}$$

where Δ is in kPa/°C, and T is the mean temperature in °C. To calculate the psychrometric constant, we must first calculate P, the atmospheric pressure that Doorenbos and Pruitt (1977) suggested could be calculated by Equation 4.32:

$$P = 101.3 - 0.01055\,H \tag{4.32}$$

where P is in kPa and H is the elevation above sea level in meters. Using P, λ calculated from Equation 4.25, and c_p, the specific heat of water at constant pressure [0.001013 kJ/kg/°C], the psychrometric constant (in kPa/°C) can be calculated from Equation 4.33:

$$\gamma = \frac{c_p\,P}{0.622\,\lambda} \tag{4.33}$$

The remaining value to calculate is G, the heat flux density to the ground in MJ/m/d, and this can be determined from Equation 4.34, knowing the mean air temperature for the time period before and after the period of interest:

$$G = 4.2\,\frac{(T_{i+1} - T_{i-1})}{\Delta t} \tag{4.34}$$

where T is the mean air temperature in °C for time period $i + 1$ and $i - 1$, and Δt is the time in days between the midpoints of time periods $i + 1$ and $i - 1$.

Crop Actual Evapotranspiration

To estimate crop actual ET (E_t):

$$E_t = k_c\,E_{tr} \quad \text{or} \quad E_t = k_c\,E_{tp} \tag{4.35}$$

where E_{tr} is reference crop ET, E_{tp} is potential ET, E_t is actual evapotranspiration and k_c is the experimentally derived crop coefficient. Typical reference crops used to develop the coefficients are alfalfa or grass.