Environmental Hydrology Chapter 3 Equations:

Soil Water Relationships, Infiltration and Soil Water Retention, Soil Water Balance, Horton Equation, Green-Ampt Equation, Darcy's Law, Richards Equation, Stokes Law, Particle Density, Hydraulic Conductivity

Soil Water Relationships

Total Volume

The total volume, V_t, is the sum of the volumes for the three phases consisting of air, water, and solids:

$$V_t = V_a + V_w + V_s \tag{3.1}$$

where the subscripts, *a*, *w*, and *s*, refer to the air, water, and solid phases. Volume can be expressed in ft^3 or cm^3 (cc) or as a depth (inches, feet, or cm) per unit cross-sectional area. For example it is not uncommon in agriculture to talk about the change in the soil water content of a soil profile as being some measured number of inches. To convert this amount into a volume it is necessary to multiply it by the surface area of the land being considered.

Porosity

Porosity is an important property in problems involving water volumes or water movement. It is commonly used in calculations made by hydrologists, soil scientists, and agricultural drainage engineers.The volume of the voids or pores is:

$$V_v = V_a + V_w \tag{3.2}$$

If we substitute Equation 3.2 into Equation 3.1, rearrange the equation, and divide the whole equation by V_t a relationship is established for the *porosity*:

$$n = \frac{V_v}{V_t} = 1 - \frac{V_s}{V_t} \tag{3.3}$$

Soil Water Content

It is often necessary to know the actual soil water content in a soil profile. Soil water contents can be expressed either by volume or by mass. The soil water content by volume, θ_v, is the volume of water in a soil sample divided by the total volume of the sample:

$$\theta_v = \frac{V_w}{V_t} \tag{3.4}$$

Normally the volumetric soil water content is expressed as a percentage and the answer from Equation 3.4 will need to be multiplied by 100. The fraction or percentage of the pores that are filled with water is called the degree of saturation. The volumetric soil water content, degree of saturation, and porosity are related as follows:

$$\theta_v = S\,n \tag{3.5}$$

where *S* is the degree of saturation, and *n* is the porosity.

Total Mass

The total mass, m_t, of a soil profile is:

$$m_t = m_a + m_w + m_s \tag{3.6}$$

where m_a is the mass of air, m_w is the mass of water, and m_s is the mass of solids. The mass of the air is negligible and the total mass is approximated as the sum of the water and soil masses.

When the soil water content is determined as a function of mass or weight, it is called the *gravimetric soil water content*, θ_g, and is equal to:

$$\theta_g = \frac{m_w}{m_s} \tag{3.7}$$

The behavior of a soil is very dependent on the *bulk density* of the soil profile. The bulk density, ρ_b, is the mass of soil per unit volume and is related to other soil physical properties as follows:

$$\rho_b = \frac{m_t}{V_t} \tag{3.8}$$

The dry bulk density is the mass of dry soil divided by the total volume:

$$\rho_{dry} = \frac{m_s}{V_t} \tag{3.9}$$

The dry bulk density of the profile can be related to the density of soil particles and porosity as follows:

$$\rho_{dry} = \rho_p (1 - n) \tag{3.10}$$

The density of the soil particles, ρ_p, is defined as the mass of dry soil divided by the volume of the soil (m_s/V_s). The density of most soil particles is 160–170 lb/ft^3 (2.6–2.75 g/cm^3). If the soil particle density is not measured, a value of 165 lb/ft^3 (2.65 g/cm^3) is often assumed.

The ratio of the density of the soil particles to the density of water is called the specific gravity:

$$G_s = \frac{\rho_p}{\rho_w} \tag{3.11}$$

where, ρ_w, the density of water is usually assumed to be 1.0 g/cm^3 or 62.4 lb/ft^3.

Infiltration and Soil Water Retention

Soil Water Tension

Soil water tension varies from less than 1 inch of water head for a soil near saturation to as much as 10,000,000 inches of head for a very dry condition. The effect of surface tension in a soil matrix during drainage can be described by considering water held in a single pore within the soil profile connected to the groundwater table. The figure describes an equilibrium state at the meniscus with radius r_1. If it is assumed that the meniscus with radius r_1 supports a column of water, h, then the gravitational forces will equal the surface tension forces and:

$$\left(\pi r_1^2\right) h \rho_w g = \left(2\pi r_1\right) \tau \cos\alpha \tag{3.12}$$

where ρ_w is the density of water, g is the gravitational constant, τ is the surface-tension force, and α is the angle of contact of the meniscus with the soil.

Plant Available Soil Water

The maximum amount of water that can be extracted from the soil by plants is termed the plant available soil water (PAW). It is related to field capacity and wilting point as follows:

$$\theta_{paw} = \theta_{fc} - \theta_{wp} \tag{3.13}$$

where θ_{paw}, θ_{fc}, and θ_{wp} are the volumetric or gravimetric plant available soil water content, the soil water contents at field capacity, and at the wilting point, respectively.

Soil Water Balance

A water balance equation describing these changes for any period of time is expressed as:

$$\Delta SM = P + IR - Q - G - ET \tag{3.14}$$

where ΔSM is the change in soil water storage in the soil profile, P is precipitation, IR is irrigation, G is percolation water, ET is evapotranspiration, and Q is surface runoff. All quantities are expressed as a depth (inches or mm) of water over a study area for a specific period of time.

Horton Equation

One of the most widely used infiltration models is the three parameter equation developed by Horton (1939):

$$f = f_c + (f_o - f_c)\, e^{-\beta t} \tag{3.15}$$

where f is the infiltration rate at time t, f_o is the infiltration rate at time zero, f_c is the final constant infiltration capacity and □ is a best fit empirical parameter. Horton's equation has seen widespread application in storm watershed models. The most commonly used model that uses Horton's method is the Environmental Protection Agency Storm Water Management Model (Huber, 1981).

Green-Ampt Equation

In 1911, Green and Ampt developed an analytical solution of the flow equation for infiltration under constant rainfall. The method is developed directly from Darcy's law and assumes a capillary-tube analogy for flow in a porous soil. The equation can be written as:

$$f = K\,(H_o + S_w + L)/L \tag{3.16}$$

where K is the hydraulic conductivity of the transmission zone, H_o is the depth of flow ponded at the surface, S_w is the effective suction at the wetting front, and L is the depth from the surface to the wetting front. The method assumes *piston flow* (water moving down as a front with no mixing) and a distinct wetting front between the infiltration zone and soil at the initial water content.

The Green-Ampt method is often approximated by the equation:

$$f = \frac{A}{F} + B \tag{3.17}$$

where f is the infiltration rate, F is the accumulative infiltration, and A and B are fitted parameters that depend on the initial soil water content, surface conditions, and soil properties.

Physically Based Methods

Darcy's Law

In 1856, Henry Darcy developed the basic relationship for describing the flow of water through a homogeneous soil. Darcy's law may be written as:

$$q = -K\frac{\partial \varphi}{\partial z} \quad (3.18)$$

where q is the flux, or volume of water moving through the soil in the *z*-direction (vertically or laterally) per unit area per unit time, and $\partial\phi/\partial z$ is the hydraulic gradient in the *z*-direction (vertically or laterally).

Richards Equation

The hydraulic conductivity is also a function of the soil water content, θ, and the pressure head, ϕ. The hydraulic conductivity is usually expressed as K(ϕ), a function of the pressure head, or as K(θ), a function of the soil water content. A theoretical differential equation for unsaturated flow is obtained by combining Darcy's equation with the continuity equation. This equation is referred to as the diffusion equation or Richards equation (Jury et al., 1991). The equation is expressed as:

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left[(D_w(\theta)\frac{\partial\theta}{\partial z}\right] + \frac{\partial K(\theta)}{\partial z} \quad (3.19)$$

where $D_w(\theta)$, the soil water diffusivity, is defined as $K(\theta)\ \partial h/\partial\theta$ and h is the matric potential or the negative pressure head. The difficulty with solving the equation is the nonlinear relationship between hydraulic conductivity, pressure head, and soil water content.

Stokes Law

The size distribution of smaller particles is found from the relationship between settling velocity and particle diameter known as Stokes' Law:

$$v = \frac{g\,(\rho_s - \rho_l)X^2}{18\,\eta} \quad (3.20)$$

where *v*, is velocity of the fall, η is fluid shear viscosity, ρ_s and ρ_l are particle and liquid densities, respectively, and *X* is the "equivalent" particle diameter.

Particle Density

The particle density methods are based on the difference in weight between a volume of water and that same volume with some of the water displaced by a known weight of soil.

The particle density can be calculated from the relationship:

$$\rho_p = \frac{\rho_w (W_s - W_a)}{(W_s - W_a) - (W_{sw} - W_w)} \tag{3.21}$$

where: ρ_p = particle density (lb/ft^3)
ρ_w = density of water (lb/ft^3)
W_s = weight of pycnometer (or weighing dish) plus oven-dry soil
W_a = weight of pycnometer (or weighing dish)
W_{sw} = weight of pycnometer (or weighing dish) filled with soil and water
W_w = weight of pycnometer (or weighing dish) filled with water

Hydraulic Conductivity

The *core sample method* determines saturated hydraulic conductivity on samples by either a constant head or falling head test. Core samples are taken in the field and kept from drying. In the laboratory, the samples are saturated from the bottom to prevent air entrapment. For the constant head test, water is supplied to the bottom of the core samples at constant hydraulic head. The volume of outflow is measured with time. The hydraulic conductivity is found by rewriting Darcy's law as:

$$K = \frac{qL}{Ah} \tag{3.22}$$

where q is the rate of outflow, L is the length of the core sample, A is the core cross-sectional area, and h is the depth of the constant head applied.

In the falling head test, used with soils of low permeability, the hydraulic conductivity is determined from the rate of change of velocity of a falling water column. A manometer (cross-sectional area **a**) is placed on top of the sample core (cross-sectional area A) and filled with water. The hydraulic conductivity is determined as:

$$K = \frac{aL}{A(t_2 - t_1)} \ln \frac{h_1}{h_2} \tag{3.23}$$

where L is the length of the soil core, h_1 is the height of the water column at time t_1, and h_2 is the height of the water column at time t_2.

In the presence of a shallow water table, the Auger hole method is the most widely used method for determining hydraulic conductivity of soils in the field. A hole is augured to the desired depth below the water table and water is allowed to rise until equilibrium is reached. The hole is then pumped or bailed and the rate of rise of the water level in the hole is measured. The saturated hydraulic conductivity is calculated as:

$$K = \frac{\pi R^2}{Sh} \frac{dh}{dt} \tag{3.24}$$

where R is the radius of the Auger hole, S is a function of hole geometry found from nomographs, h is the depth of water in the Auger hole, and dh/dt is the rise in the water level over time increment dt.