Environmental Hydrology Chapter 13 Equations:

Projected Source Area, Solid Angle, Radiance, Spectral Relative Units

Projected Source Area

The area of the earth surface measured by the sensor will change depending on the angle that the sensor views the feature. This can be addressed by taking the Cosine of the area of a plate of material or feature on the ground. This corrects for the geometry of the viewer or sensor at some other position than directly beneath the sensor or the "nadir" position. The area that is viewed by the remote sensor is equal to A Cos 2.

$$L = \frac{\rho\, E(\text{on a feature})}{\pi\, A \cos\theta}\left(\frac{a}{R^2}\right) \tag{13.1}$$

where L is the radiance, E is the irradiance, a is the area of the aperture, R is the distance between the feature and the sensor, ρ is the reflectance, θ is the angle from the perpendicular that the sensor views the feature, and A is the area of the feature on the surface being imaged.

Solid Angle

One must address the geometry of measurements. This is particularly important when light is radiated into and through a three-dimensional material such as water or the atmosphere. Here, the concept of solid angle is employed. For three-dimensional or volumetric cases the concept of the steradian is most appropriate. The solid angle, ω (omega), is subtended by an area on the surface of a sphere that is equal to the square of the radius of the sphere, r_s, and the a is the area of the entrance aperature of a sensor.

$$\omega = \frac{a}{r_s^2} \tag{13.2}$$

The total solid angle about an isotrophic point feature is:

$$\omega = 4\pi \frac{r_s^2}{r_s^2} = 4\pi \tag{13.3}$$

It is common to cite the resolution of a sensor as the radians of the cone angle of the sensor. The sensor records information on the ground, e.g., 0.5 milliradians. This is the angle of the cone viewed by the sensor. An airborne sensor can fly at different altitudes, and hence may have different ground resolutions depending on the height above ground.

To determine the solid angle we use the equation:

$$\omega = \pi \tan^2 \frac{\theta}{2} \tag{13.4}$$

When provided with the cone angle in radians, we can calculate any other portion of the cone using the relationship:

$$\tan \frac{\theta}{2} = \frac{r_s}{2} \tag{13.5}$$

2/2 occurs because the entire cone angle is provided.

Radiance

Radiance (L) is equal to the irradiance (E) divide by the solid angle, or E/ω or E/π. The units are Watts/m^2/sr. The radiance, L, of a feature imaged or measured at a distance or altitude above the surface or earth, "R", by the sensor is:

$$L_{\text{at sensor}} = \frac{E(\text{on surface})}{\pi} (\cos\theta) \left(\frac{a}{R^2} \right) \tag{13.6}$$

$$L_{\text{at sensor}} (\text{off surface}) x (\text{viewing surface}) = \frac{M(\text{off surface})}{\pi} \left(\frac{a}{R^2} \right) \tag{13.7}$$

where "a" is the area of the opening or aperture of the camera or sensor, R is the distance from the feature to the sensor or the altitude above the surface or ground level (AGL), and "M" is the existence or irradiance off the surface or earth (Watts/m^2/sr, the same as "E" but in the upwelling direction).

The actual quantity of light measured by the sensor is therefore a function of distance, and a function of the size area of the opening or aperture in the sensor, "a". Hence, a/R^2 accounts for the geometry of aerial or space measurements, and the radiance at the sensor:

$$L_{\text{at sensor}} = \frac{E(\text{off surface})}{\pi} \left(\frac{a}{R^2} \right) \tag{13.8}$$

A correction can be made for measuring features away from the nadir of the sensor, or from features that have a slope angle:

$$L_{sensor} = \frac{E(\text{off surface})}{\pi}(A\cos\theta)\left(\frac{a}{R^2}\right) \quad (13.9)$$

where 2 is the angle from which the sensor views the feature away from the normal vector view to the feature or ground surface. Use of a/R^2 assumes that we are dealing with a point source. This is a good assumption if the distance or altitude, R, is at least five to ten times the radius of the entrance aperture. For this case any influence due to geometry is minimal and is ignored.

Spectral Relative Units

Typical dimensions are spectral radiance or 8 = Watts/(m^2·sr·nm), or Watts/(m^2·sr·um). For example, the flux or irradiance, i, in a narrow band is the value of the spectral flux at the interval center times the bandwidth interval. The sum of the fluxes in an adjacent narrow bandwidth provides the flux of the bandwidth.

$$\sum i = \sum[(\lambda)(\partial\lambda_i)] \quad (13.10)$$

To obtain the spectral response of a given material in a certain bandwidth of operation, the area under a given spectral response curve must be integrated. This allows the modeling of spectral response of materials or combinations of material using existing spectral response curves from references. There are several methods to integrate the area under the curve within the wavelength region of operation. A simple approach is the Trapezoidal Rule:

$$\text{area under curve} = \Delta\lambda\,(\text{nm})\left(\frac{x_1}{2} + x_1 + x_3 + \ldots\ldots x_{n-1} + \frac{x_n}{2}\right) \quad (13.11)$$

For example, we can make a sample calculation using spectral flux measurements taken from a graph at the mid-points of the 20 nm intervals, and using the Trapezoidal rule, with) 8 = 20 nm (Flux of 400 to 600 nm):

$$\text{Flux(400 to 600 nm)} = 20\text{nm}\left(\frac{1.2}{2} + 1.1 + 1.0 + \ldots x_{n-1} + \frac{x_n}{2}\frac{W}{nm}\right) \quad (13.12)$$

$$= 20\text{nm}\left(\frac{1.2}{2} + 1.1 + 1.0 + 1.2 + 1.3 + 1.0 + 1.1 + 1.4 + 1.1 + 1.2 + \frac{1.2}{2}\frac{W}{nm}\right)$$

$$\text{Flux(400 to 600 nm)} = 20\text{nm}\left(11.6\frac{W}{nm}\right) = 232\text{W}$$

In many applications using measurements or calculations it is desirable to work in relative units. The relative units are usually simple ratios of some absolute measures of incoming radiation and some portion of the outgoing radiation. A common example of relative units is reflectance.

Radiation that is retained within the material is the absorbance component, such as light absorbed by plankton in water.

The general equations for these relative units are:

$$\text{Reflectance} = \rho = \frac{\text{radiation off of surface of materials}}{\text{radiation incident on materials}} \tag{13.13}$$

$$\text{Transmittance} = \tau = \frac{\text{radiation through the materials}}{\text{radiation incident on materials}} \tag{13.14}$$

$$\text{Absorbance} = \alpha = \frac{\text{radiation absorbed within the materials}}{\text{radiation incident on materials}} \tag{13.15}$$

The relative units are calculated as ratios of the radiometric units measured or calculated. Commonly, we will ratio irradiances, or radiances, though any two equivalent units can be employed. Use of the above measurement and modeling approaches allows the evaluation of remote sensor experiments. These elements can be combined to study the response of materials to light energy, and to estimate the quantity of light that can be measured by remote sensors during experiments.