Environmental Hydrology Chapter 11 Equations:

Groundwater Flow, Flow to Wells, Capture Zones of Wells

Groundwater Flow

Darcy's Equations

From his experiments Darcy observed that if the flow rate, Q was doubled, the loss of head ($dh = h_1 - h_2$) between manometers X and Y also doubled. Thus, Q is directly proportional to head loss, dh and the hydraulic gradient, dh/l. Mathematically this is expressed as:

$$Q \approx \frac{h_1 - h_2}{l} = \frac{dh}{l} \tag{11.1}$$

Based on this relation, it can be seen that it takes twice as much energy to drive the water through the sand at twice the flow rate. If a different cylinder is used, one in which the cross-sectional area of flow is twice as large as in the first cylinder, for a particular grade of sand, twice as much water will flow through the larger cylinder than through the smaller cylinder. Thus:

$$Q \approx A \tag{11.2}$$

Combining these results, the following relation is obtained:

$$Q \approx A \frac{dh}{l} \tag{11.3}$$

Darcy repeated his experiments using several different grades of sand and found that for a given grade of sand:

$$Q = (K)(A)\left(\frac{dh}{l}\right) \tag{11.4}$$

where K is a constant of proportionality for a given grade of sand. The experiments showed that the value of K was larger for a coarse sand than for a fine sand. Darcy deduced that the value of K was related to the ability of the sand to transmit fluid. Thus, K is referred to as hydraulic conductivity and represents the properties of the porous material and the properties of the fluid. (Intrinsic permeability is a term used to describe the ability of a porous material to transmit fluid, but refers solely to the properties of the porous material.) The units of K are length per time and are commonly expressed as feet per day or centimeters per second. Although Darcy did not perform any experiments using fluids other than water, it can be visualized that a more viscous fluid would flow more slowly through the column of sand than a less viscous fluid.

The form of Darcy's Law written in Equation 11.4 can be expanded to separate the transmitting properties of the porous material from the properties of the fluid such that:

$$Q = (k)\left(\frac{\gamma}{\mu}\right)(A)\left(\frac{dh}{l}\right) \tag{11.5}$$

where k is the intrinsic permeability of the porous material, γ is the specific weight of the fluid, and μ is the dynamic viscosity of the fluid. Thus, the rate of fluid flow is directly proportional to the specific weight of the fluid and inversely proportional to its viscosity.

Transmissivity

Another way to express the ability of a porous material to transmit water is to account for its transmission across the entire saturated thickness of the permeable materials. Transmissivity is expressed in terms of length squared per time and is equal to:

$$T = Kb \tag{11.6}$$

Figure 11.6 shows a bar graph for estimating the capability of porous materials to transmit water to wells based on the measured transmissivity of the materials. This graph is especially useful in estimating whether a particular geologic material is capable of transmitting sufficient water to various types of wells.

Specific Yield

Values of *specific yield* (Sy) commonly are expressed as a percentage. Sy values are less than the porosity of the aquifer materials and range from less than 1% to as much as 45%. The porosity (n), also commonly expressed as a percentage, of a soil or rock is defined as the volume of void space relative to the total volume. The difference between n and Sy is the volume of groundwater that does not drain under the influence of gravity, which is called the specific retention (Sr). Thus:

$$n = Sy + Sr \tag{11.7}$$

Grain-size distribution and grain shape are the primary factors controlling the distribution of pore sizes and the rate of gravity drainage.

Storage

Seasonal water-level fluctuations in confined aquifers also reflect changes in the amount of water in storage but not by the same mechanism as for unconfined aquifers. As potentiometric levels in a confined aquifer change, the pores remain saturated. The change in potentiometric level represents a change in the amount of pressure head in the aquifer. The amount of water-level change is controlled by the coefficient of storage (S), which is defined as:

$$S = \rho g b(\alpha + n\beta) \tag{11.8}$$

where ρ is the density of the fluid, g is the gravitational constant, b is the saturated thickness of the aquifer, α is the compressibility of the aquifer skeleton, n is the porosity, and β is the compressibility of the fluid. Values of S are small, on the order of 0.0001 to 0.000001. Thus, it may require 1000 times the volume of soil/rock to store the same amount of groundwater in a confined as in an unconfined aquifer.

Average Linear Velocity of Groundwater Flow

Differences in the rate of infiltration of water through the unsaturated zone can cause differences in the rate at which recharge from precipitation reaches the water table, creating nonuniform rates of water-level rise. This results in changes in hydraulic gradient that are manifest as changes in the rate and direction of groundwater flow. The rate of groundwater flow is computed using Darcy's Law and accounts for the fact that flow occurs only through the pores. Thus, the average linear velocity (v) of groundwater flow is:

$$v = \frac{K\frac{dh}{l}}{n_e} \tag{11.9}$$

where n_e is the effective porosity, expressed as a decimal, and is defined as the volume of interconnected pore space relative to the total volume.

Flow to Wells

Mass Balance Using Darcy's Law

The response of a groundwater flow system to the stress created by pumping a well can be conceptualized using Darcy's Law. Assuming the response of the flow system to the pumping stress is steady-state and that the well discharges at a constant rate Q, then the quantity of water passing across the circumferential area represented by the larger cylinder must he equal to the quantity of water passing across the circumferential area represented by the smaller cylinder which must be equal to the pumping rate of the well. This mass balance can be expressed by Darcy's Law such that:

$$Q = (K)(2\pi r_1 b)\left(\frac{dh_1}{dr_1}\right) = (K)(2\pi r_2 b)\left(\frac{dh_2}{dr_2}\right) \tag{11.10}$$

where dh_1/dr_1 and dh_2/dr_2 are the hydraulic gradients at r_1 and r_2, and $2\pi r_1 b$ and $2\pi r_2 b$ are the circumferential areas at r_1 and r_2. Cancelling terms and recognizing that if $r_1 < r_2$, then $dh_1/dr_1 > dh_2/dr_2$. Thus, as the distance to the pumping well becomes smaller and smaller, the hydraulic gradient becomes steeper and steeper, forming a cone of depression about the well.

Theis Equation

The well is assumed to fully penetrate the aquifer, to discharge at a constant rate, and to have a negligible amount of water stored in the borehole. Under these conditions the Theis equation can be used to compute the drawdown at any radial distance from the pumped well at any time, such that

$$s = \frac{Q}{4\pi T} W(u) \tag{11.11}$$

and

$$u = \frac{r^2 S}{4Tt} \tag{11.12}$$

where *s* is drawdown, *Q* is the pumping rate, *T* is transmissivity, *t* is time, *S* is the coefficient of storage, *r* is the radial distance to a point of interest, and *W*(*u*) is the well function of *u*. Any set of consistent units can be used in these equations. The value of the well function can be computed using a series expansion such that:

$$W(u) = \left[-0.5772 - \ln u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} + \cdots \right] \tag{11.13}$$

Values of *W*(*u*) for various values of *u* are listed in Table 11.1 or can be computed using a spreadsheet program and expanding the well function to 9–10 terms.

In many problems it is necessary to account for the drawdowns produced by several wells. This is a common situation in the design of municipal wellfields, irrigation wells, contaminant pump-and-treat systems, and construction of dewatering/depressurizing systems for excavations. Because drawdowns are additive, the mathematical principle of superposition can be used to address these more complex problems. Thus, the composite drawdown from any number of wells at a particular location at a specific time is equal to the sum of the drawdowns from each individual well. Mathematically, this can be expressed simply as:

$$s_{total} = s_1 + s_2 + s_3 + \cdots. \tag{11.14}$$

where the subscripts refer to the component of the total drawdown due to an individual well. If the Theis assumptions are met, the following equation would apply to the calculation of the composite drawdown at a location between two pumping wells discharging at different rates:

$$s = \frac{Q_1}{4\pi T} W(u_1) + \frac{Q_2}{4\pi T} W(u_2) \tag{11.15}$$

where

$$u_1 = \frac{r_1^2 S}{4Tt} \quad \text{and} \quad u_2 = \frac{r_2^2 S}{4Tt}$$

The assumptions required of the Theis equation are not met in all situations. In many geologic settings, aquifers are not infinite but are bounded on one or more sides by streams, which can provide an additional source of water in unconfined aquifers, or by bedrock valley walls or faults, which can preclude the flow of water. In an unconfined aquifer, the Theis equation will not make accurate predictions of the amount of drawdown near wells because the assumption of constant aquifer thickness is violated by the formation of the cone of depression in the water table around the well. (This is not a problem in confined aquifers because the cone of depression is formed in the potentiometric surface above the base of the overlying confining layer.) In the case of an unconfined aquifer, solution of the following equation using the quadratic formula enables drawdowns computed with the Theis equation to be corrected for the decrease in saturated thickness near the well and applied to an unconfined aquifer:

$$s^2 - 2bs = 2bs' \tag{11.16}$$

where *s* is uncorrected drawdown computed using the Theis equation for a confined aquifer, *b* is the initial saturated thickness of the aquifer prior to pumping, and *s′* is the corrected drawdown in an equivalent unconfined aquifer. This correction factor is important near pumping wells but becomes increasingly less important with distance from the well. When the correction factor is less than 0.01 ft, it is below the precision of most methods of measuring hydraulic head and, therefore, can be ignored.

Capture Zones of Wells

The capture zone of a well extends upgradient to a regional groundwater divide. It extends downgradient only as far as a local groundwater divide created by the cone of depression of the well. In a Cartesian coordinate system, the bounding flowline within which all groundwater flows to the well can be estimated with the following equation:

$$x = \frac{-y}{\tan\left[\dfrac{2\pi Kbiy}{Q}\right]} \tag{11.17}$$

where *x* and *y* are coordinate values, *Q* is the pumping rate of the well, *K* is the hydraulic conductivity of the aquifer, *b* is the saturated thickness of the aquifer, *i* is the regional (pre-pumping) hydraulic gradient, and tan [ ] is in radians.

The distance to the downgradient groundwater divide created by the well is given by:

$$x_0 = \frac{-Q}{2\pi Kbi} \tag{11.18}$$

The maximum half-width of the capture zone as *x* approaches infinity is given by:

$$y_{max} = \pm\frac{Q}{2Kbi} \tag{11.19}$$

From these equations it can be seen that the distance to the downgradient groundwater divide is proportional to the pumping rate but inversely proportional to the hydraulic gradient and *K*. Thus, the greater the pumping rate or the flatter the regional hydraulic gradient, the greater the distance to the downgradient groundwater divide. These equations also show that the width of the capture zone is proportional to the pumping rate but inversely proportional to the hydraulic gradient and *K*. Thus, the greater the pumping rate or the smaller the regional hydraulic gradient, the wider the capture zone.